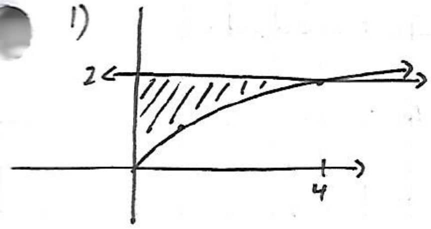
AP Calculus AB Volumes of Solids

1)



a) x-axis - dx
WASHER
$R = 2 \quad r = \sqrt{x}$
$V = \pi\int_0^4 4\,dx - \pi\int_0^4 \sqrt{x}^{\,2}\,dx$
$= \pi\int_0^4 4\,dx - \pi\int_0^4 x\,dx$
$= 16\pi - \pi\left[\frac{1}{2}x^2\right]_0^4$
$= \boxed{8\pi}$

b) y-axis - dy
DISC $\quad y = \sqrt{x} \rightarrow x = y^2$
$R = y^2$
$V = \pi\int_0^2 y^4\,dy$
$= \pi\left[\frac{1}{5}y^5\right]_0^2$
$= \boxed{\frac{32\pi}{5}}$

c) $y = 2$ - dx
Disc
$R = 2 - \sqrt{x}$
$V = \pi\int_0^4 (2-\sqrt{x})^2\,dx$
$= \pi\int_0^4 (4 - 4x^{1/2} + x)\,dx$
$= \pi\left[4x - \frac{8}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^4$
$= \pi\left[16 - \frac{64}{3} + 8\right]$
$= \frac{8\pi}{3}$

d) $x = 4$ - dy
WASHER
$R = 4 \quad r = 4 - y^2$
$V = \pi\int_0^2 4\,dy - \pi\int_0^2 (4-y^2)^2\,dy$
$= 8\pi - \pi\int_0^2 (16 - 8y^2 + y^4)\,dy$
$= 8\pi - \pi\left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right]_0^2$
$= 8\pi - \pi\left[32 - \frac{64}{3} + \frac{32}{5}\right]$

2) $y = \sqrt{x} \quad y = \frac{1}{2}x$

a) Area $= \int_0^4 \left(x^{1/2} - \frac{1}{2}x\right)dx$
$= \left[\frac{2}{3}x^{3/2} - \frac{1}{4}x^2\right]_0^4$
$= \frac{16}{3} - 4$
$= 4/3$

b) $y = 2$ - dx
WASHER
$V = \pi\int_0^4 R^2\,dx - \pi\int_0^4 r^2\,dx$
$R = 2 - \frac{1}{2}x \quad r = 2 - \sqrt{x}$

c) $V = \int_0^4 (\text{side})^2\,dx$
side $= \sqrt{x} - \frac{1}{2}x$

3) a) $\int_0^9 g(x)\,dx$

$\int_0^4 g(x)\,dx + \int_4^9 g(x)\,dx$

$8 \; + -2 \; = \boxed{6}$

b) $\int_4^9 f(x)\,dx$

$\int_0^4 f(x)\,dx + \int_4^9 f(x)\,dx = \int_0^9 f(x)\,dx$

$7 \; + \int_4^9 f(x)\,dx = 12$

$\int_4^9 f(x)\,dx = \boxed{5}$

c) $\int_9^4 g(x)\,dx = -\int_4^9 g(x)\,dx$

$= \boxed{2}$

d) $\int_4^9 \left[f(x) + 3g(x)\right] dx$

$\int_4^9 f(x)\,dx \; + 3\int_4^9 g(x)\,dx$

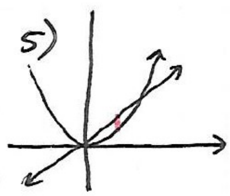
$5 \; + 3(-2) \; = \boxed{-1}$

e) $\int_9^9 g(x)\,dx = 0$

4) $\lim\limits_{x\to 0} \dfrac{\sin 4x}{2x} \to \dfrac{0}{0}$

using L'H's Rule

$\lim\limits_{x\to 0} \dfrac{4\cos 4x}{2} = 2$

5) 

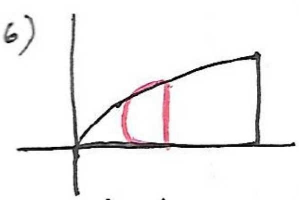
$V = \int_0^1 (\text{side})^2\,dx$

$= \int_0^1 (x - x^2)^2\,dx$

$= \int_0^1 (x^2 - 2x^3 + x^4)\,dx$

$= \left[\frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5\right]_0^1$

$= \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \; = \boxed{\frac{1}{30}}$

6) 

Semicircles

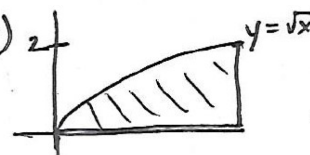
$V = \frac{\pi}{8}\int_0^4 \text{diameter}^2$

$= \frac{\pi}{8}\int_0^4 (\sqrt{x})^2\,dx$

$= \frac{\pi}{8}\int_0^4 x\,dx$

$= \frac{\pi}{8}\left[\frac{1}{2}x^2\right]_0^4$

$= \boxed{\pi}$

7) 

y-axis - dy

WASHER

$y = \sqrt{x} \to x = y^2$

$V = \pi\int_0^2 R^2\,dy - \pi\int_0^2 r^2\,dy$

$R = 4 \qquad r = y^2$

$= \pi\int_0^2 16\,dy - \pi\int_0^2 y^4\,dy$

$= 32\pi - \pi\left[\frac{1}{5}y^5\right]_0^2$

$= 32\pi - \frac{32\pi}{5} = \boxed{\frac{128\pi}{5}}$